Prediction for Quark Mixing from Universal Quark and Lepton Mass Matrices with Flavor 2 ↔ 3 Symmetry

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Abstract

We propose a mass matrix model that gives a unified description of quark and lepton with the same texture form based on a flavor 2 ↔ 3 symmetry. The model is in contrast with the conventional picture that the mass matrix forms in the quark sector will take somewhat different structures from those in the lepton sector. By investigating possible types of assignment for masses, we find that our quark mass matrices can lead to small CKM quark mixings which are consistent with experimental values. We also find some phase-parameter-independent relations among CKM quark mixing matrix elements.

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I. INTRODUCTION

Recent neutrino oscillation experiments [1] have highly suggested a nearly bimaximal lepton mixing ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \simeq 1$) in contrast with the small quark mixing. In order to reproduce these large lepton mixing and small quark mixing, mass matrices of various structures with texture zeros have been investigated in the literature [2]-[4]. Recently a following mass matrix model based on a discrete symmetry Z_3 and a flavor $2 \leftrightarrow 3$ symmetry has been proposed [5] for all quarks and leptons:

$$M = P^\dagger \begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & B \end{pmatrix} P^\dagger, \tag{1.1}$$

where A, B, and C are real parameters. The diagonal phase matrix P which breaks $2 \leftrightarrow 3$ symmetry has been introduced from a phomelogical points of view. This structure of mass matrix has been previously suggested and used for the neutrino mass matrix in Refs. [6]–[11] motivated by the experimental finding of maximal ν_μ–ν_τ mixing [1]. We consider that this structure is fundamental for both quarks and leptons, although it was speculated from the neutrino sector. Therefore, we assume that all the mass matrices have this structure, which is against the conventional picture that the mass matrix forms in the quark sector will take somewhat different structures from those in the lepton sector. In our previous works [5, 14, 15], we have pointed out that this structure leads to reasonable values for the Cabibbo–Kobayashi–Maskawa (CKM) [12] quark mixing as well as large lepton mixing. However, the diagonal phase matrix P has been introduced artificially into the model.

Recently, a new way to include phases into the mass matrix M has been proposed by Koide [16] as

$$M = \begin{pmatrix} 0 & ae^{-i\phi} & a \\ ae^{-i\phi} & be^{-2i\phi} & (1-\xi)b \\ a & (1-\xi)b & b \end{pmatrix}, \tag{1.2}$$

where a, b, and ξ are real parameters and ϕ is a phase. M is invarient under the "extended" flavor $2 \leftrightarrow 3$ permutation T_{23}^ϕ for the fields:

$$T_{23}^\phi M \left(T_{23}^\phi\right)^T = M, \tag{1.3}$$

with

$$T_{23}^{\phi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{-i\phi} \\ 0 & e^{i\phi} & 0 \end{pmatrix}. \tag{1.4}$$

Koide [16] has discussed the democratic case ($\xi = 0$) and obtained CKM mixing matrix which is roughly in agreement with experimental values. However, $|V_{cb}|$ is somewhat smaller than the observed value.

In the present paper, extending the Koide's model, we propose a nonsymmetric and nondemocratic mass matrix by adding a new parameter a' to reproduce a consistent value for $|V_{cb}|$. Our mass matrix form is

$$M = \begin{pmatrix} 0 & ae^{-i\phi} & a \\ a'e^{-i\phi} & be^{-2i\phi} & (1-\xi)b \\ a' & (1-\xi)b & b \end{pmatrix}. \tag{1.5}$$

In order to avoid difficulties raised in Ref [17] in considering flavor symmetry, we consider that the ϕ is a phase which breaks the $2 \leftrightarrow 3$ symmetry, although it is originally introduced by the "extended" $2 \leftrightarrow 3$ symmetry. In this paper, we shall investigate CKM quark mixing from the phenomenological point of view by using the mass matrix given in Eq. (1.5).

This article is organized as follows. In Sec. II, we discuss the mass matrix of our model. The numerical values of CKM quark mixing matrix elements at the unification scale are obtained from the observed values at the electroweak scale in Sec. III. The quark mixing matrix in the present model is argued in Sec. IV. Sec. V is devoted to a summary.

II. MASS MATRIX

Our mass matrices M_u, M_d, M_ν, and M_e for up quarks (u, c, t), down quarks (d, s, b), neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and charged leptons (e, μ, τ), respectively are given as follows:

$$M_f = \begin{pmatrix} 0 & a_f e^{-i\phi_f} & a_f \\ a'_f e^{-i\phi_f} & b_f e^{-2i\phi_f} & (1-\xi_f)b_f \\ a'_f & (1-\xi_f)b_f & b_f \end{pmatrix}, \quad (f = u, d, \nu, \text{ and } e) \tag{2.1}$$

where a_f, b_f, ξ_f, and a'_f are real parameters and ϕ_f is a phase parameter. Here we consider a nonsymmetric mass matrix based on the flavor $2 \leftrightarrow 3$ symmetry with including

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one symmetry-breaking phase parameter. From Eq. (2.1), hermitian matrix $M_f M_f^\dagger$ can be decomposed to following two parts:

$$M_f M_f^\dagger = 2a_f^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + P_f^\dagger \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} P_f, \tag{2.2}$$

where real components A_f, B_f, and C_f are given by

$$A_f = a_f |X_f|, \tag{2.3}$$

$$B_f = (a_f')^2 - 2a_f^2 + b_f^2 + (1 - \xi_f)^2 b_f^2, \tag{2.4}$$

$$C_f = (a_f')^2 + (1 - \xi_f)b_f^2(e^{i\phi_f} + e^{-i\phi_f}), \tag{2.5}$$

and the diagonal phase matrix P_f is given by

$$P_f = \text{diag}\left(1, e^{i(\phi_f - \varphi_f)}, e^{-i\varphi_f}\right). \tag{2.6}$$

Here we have defined $|X_f|$ and phase φ_f as the absolute and phase parts of $X_f \equiv b_f + (1 - \xi_f)b_f e^{i\phi_f}$, respectively:

$$X_f \equiv b_f + (1 - \xi_f)b_f e^{i\phi_f} \equiv |X_f|e^{i\varphi_f}. \tag{2.7}$$

First we discuss three mass eigenvalues m_{if} (i=1,2, and 3) of M_f. From Eq. (2.2), the eigenvalues m_{if}^2 of $M_f M_f^\dagger$ are obtained in terms of the components as

$$m_{1f}^2 = \frac{1}{2}\left[B_f + C_f - \sqrt{8A_f^2 + (B_f + C_f)^2}\right] + 2a_f^2, \tag{2.8}$$

$$m_{2f}^2 = \frac{1}{2}\left[B_f + C_f + \sqrt{8A_f^2 + (B_f + C_f)^2}\right] + 2a_f^2, \tag{2.9}$$

$$m_{3f}^2 = (B_f - C_f) + 2a_f^2, \tag{2.10}$$

from which, by using Eqs.(2.3)-(2.5), we obtain

$$m_{1f} = \frac{1}{2}\left[\sqrt{|X_f|^2 + 2(a_f' - a_f)^2} - \sqrt{|X_f|^2 + 2(a_f' + a_f)^2}\right] < 0, \tag{2.11}$$

$$m_{2f} = \frac{1}{2}\left[\sqrt{|X_f|^2 + 2(a_f' - a_f)^2} + \sqrt{|X_f|^2 + 2(a_f' + a_f)^2}\right] > 0, \tag{2.12}$$

$$m_{3f} = \sqrt{b_f^2 + (1 - \xi_f)^2 b_f^2 - 2(1 - \xi_f)b_f^2 \cos\phi_f} > 0. \tag{2.13}$$

There are five parameters, a_f, a_f', b_f, ξ_f, and ϕ_f in M_f. If we fix above three eigenvalues m_{if} by the quark masses, we have only two free parameters. As the free parameters, let us

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chose a parameter α_f defined by

$$\alpha_f \equiv \frac{a'_f}{a_f}, \tag{2.14}$$

and a phase parameter η_f defined in Fig. 1 independently of mass eigenvalues m_{if}. Then, from Eqs. (2.11), (2.12), and (2.14), the parameters a_f and $|X_f|$ are expressed in term of α_f as

$$a_f^2 = \frac{|m_{1f}|m_{2f}}{2\alpha_f}, \tag{2.15}$$

$$|X_f|^2 = m_{1f}^2 + m_{2f}^2 - |m_{1f}|m_{2f}\left(\frac{1+\alpha_f^2}{\alpha_f}\right)$$

$$= -\frac{|m_{1f}|m_{2f}}{\alpha_f}\left(\alpha_f - \frac{|m_{1f}|}{m_{2f}}\right)\left(\alpha_f - \frac{m_{2f}}{|m_{1f}|}\right). \tag{2.16}$$

From Fig. 1 with Eqs. (2.7) and (2.13), the parameters φ_f, ϕ_f, b_f, and $(1-\xi_f)b_f$ are expressed by $|X_f|$ and η_f as

$$\cos\varphi_f = \frac{|X_f| - m_{3f}\cos\eta_f}{\sqrt{|X_f|^2 + m_{3f}^2 - 2m_{3f}|X_f|\cos\eta_f}}, \tag{2.17}$$

$$\cos(\phi_f - \varphi_f) = \frac{|X_f| + m_{3f}\cos\eta_f}{\sqrt{|X_f|^2 + m_{3f}^2 + 2m_{3f}|X_f|\cos\eta_f}}, \tag{2.18}$$

$$\cos\phi_f = \frac{|X_f|^2 - m_{3f}^2}{\sqrt{\left(|X_f|^2 + m_{3f}^2\right)^2 - 4m_{3f}^2|X_f|^2\cos^2\eta_f}}, \tag{2.19}$$

$$b_f^2 = \frac{1}{4}\left(|X_f|^2 + m_{3f}^2 - 2|X_f|m_{3f}\cos\eta_f\right), \tag{2.20}$$

$$(1-\xi_f)^2 b_f^2 = \frac{1}{4}\left(|X_f|^2 + m_{3f}^2 + 2|X_f|m_{3f}\cos\eta_f\right). \tag{2.21}$$

Therefore, by using Eqs. (2.15) - (2.21), the all texture components in M_f are given by α_f and η_f. It should be noted that from Eq. (2.16) we have the bound for the parameter α_f as

$$\frac{|m_{1f}|}{m_{2f}} \le \alpha_f \le \frac{m_{2f}}{|m_{1f}|}, \tag{2.22}$$

and also $|X_f|$ has an upper bound as

$$0 \le |X_f| \le m_{2f} - |m_{1f}|. \tag{2.23}$$

Secondly we discuss an unitary matrix U_{Lf} which diagonalizes $M_f M_f^\dagger$. The explicit expression of U_{Lf} depends on the following three types of assignment for m_i as shown in

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the Appendix:

(i) Type A:

In this type, the mass eigenvalues $|m_{1f}|$, m_{2f}, and m_{3f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $|m_{1f}| \ll m_{2f} \ll m_{3f}$.) In this type, the $M_f M_f^\dagger$ in Eq. (2.2) is diagonalized by an unitary matrix U_{Lf} as

$$U_{Lf}^\dagger M_f M_f^\dagger U_{Lf} = \text{diag}\left(m_{1f}^2, m_{2f}^2, m_{3f}^2\right), \tag{2.24}$$

where

$$U_{Lf} = P_f^\dagger O_f. \tag{2.25}$$

Here P_f is the diagonal phase matrix given in Eq. (2.6) and O_f is the orthogonal matrix which diagonalizes the real symmetric matrix in the second term in Eq. (2.2):

$$O_f^T \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} O_f = \text{diag}(-\lambda_{1f}, \lambda_{2f}, \lambda_{3f}). \tag{2.26}$$

Here, the eigenvalues λ_{1f}, λ_{2f}, and λ_{3f} are given by

$$-\lambda_{1f} = m_{1f}^2 - 2a_f^2 = m_{1f}^2 - \frac{|m_{1f}|m_{2f}}{\alpha_f}, \tag{2.27}$$

$$\lambda_{2f} = m_{2f}^2 - 2a_f^2 = m_{2f}^2 - \frac{|m_{1f}|m_{2f}}{\alpha_f}, \tag{2.28}$$

$$\lambda_{3f} = m_{3f}^2 - 2a_f^2 = m_{3f}^2 - \frac{|m_{1f}|m_{2f}}{\alpha_f}, \tag{2.29}$$

and the orthogonal matrix O_f is expressed as

$$O_f \equiv \begin{pmatrix} c_f & s_f & 0 \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.30}$$

where

$$c_f = \sqrt{\frac{\lambda_{2f}}{\lambda_{2f} + \lambda_{1f}}} = \sqrt{\frac{m_{2f}^2 - \frac{|m_{1f}|m_{2f}}{\alpha_f}}{m_{2f}^2 - m_{1f}^2}}, \tag{2.31}$$

$$s_f = \sqrt{\frac{\lambda_{1f}}{\lambda_{2f} + \lambda_{1f}}} = \sqrt{\frac{\frac{|m_{1f}|m_{2f}}{\alpha_f} - m_{1f}^2}{m_{2f}^2 - m_{1f}^2}}. \tag{2.32}$$

It should be noted that the mixing angles are functions of only α_f, since the m_{if} is fixed by the experimental quark mass values. We find from Eq. (2.19) that $\phi_f \approx \pm\pi$ for $m_{1f}^2 \ll m_{2f}^2 \ll m_{3f}^2$ in this type A assignment.

(ii) Type B:

In this type, the mass eigenvalues $|m_{1f}|$, m_{3f}, and m_{2f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $|m_{1f}| \ll m_{3f} \ll m_{2f}$.) The real symmetric matrix in the second term in Eq. (2.2) is diagonalized by an orthogonal matrix O_f' as follows:

$$O_f'^T \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} O_f' = \text{diag}(-\lambda_{1f}, \lambda_{3f}, \lambda_{2f}). \tag{2.33}$$

Here O_f' is obtained from O_f by exchanging the second row for the third one as

$$O_f' \equiv \begin{pmatrix} c_f & 0 & s_f \\ -\frac{s_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}. \tag{2.34}$$

Therefore the $M_f M_f^\dagger$ in Eq. (2.2) is diagonalized by an unitary matrix U_{Lf} as

$$U_{Lf}^\dagger M_f M_f^\dagger U_{Lf} = \text{diag}\left(m_{1f}^2, m_{3f}^2, m_{2f}^2\right), \tag{2.35}$$

where

$$U_{Lf} = P_f^\dagger O_f'. \tag{2.36}$$

In this type B assignment, we find $\phi_f \sim 0$ which is in contrast with the case in type A.

(iii) Type C:

In this type, the mass eigenvalues m_{3f}, $|m_{1f}|$, and m_{2f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $m_{3f} \ll |m_{1f}| \ll m_{2f}$.) In this type, we have

$$O_f''^T \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} O_f'' = \text{diag}(\lambda_{3f}, -\lambda_{1f}, \lambda_{2f}), \tag{2.37}$$

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$$U_{Lf}^\dagger M_f M_f^\dagger U_{Lf} = \text{diag}\left(m_{3f}^2, m_{1f}^2, m_{2f}^2\right), \tag{2.38}$$

where

$$U_{Lf} = P_f^\dagger O_f''. \tag{2.39}$$

Here, the orthogonal matrix O_f'' is given by

$$O_f'' \equiv \begin{pmatrix} 0 & c_f & s_f \\ \frac{1}{\sqrt{2}} & -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}. \tag{2.40}$$

This type is not so useful to get the reasonable CKM quark mixing values.

III. ESTIMATION OF THE MIXING ANGLES AT $\mu = M_X$

The mass matrix in Eq. (2.1) holds only at the unification scale $\mu = M_X$. Therefore, in order to compare our model and the experimental data, we calculate numerical values of the CKM mixing matrix elements at $\mu = M_X$ from their observed values at the electroweak scale $\mu = m_Z$. We estimate the evolution effects for the CKM matrix elements from $\mu = m_Z$ to $\mu = M_X$ by using the two-loop renormalization group equation (RGE). In the numerical calculations, we use the running quark masses which is estimated in Ref. [18] [minimal supersymmetric standard model with tanβ=10 case] :

$$
\begin{aligned}
m_u(m_Z) &= 2.33^{+0.42}_{-0.45}\,\text{MeV}, \quad m_c(m_Z) = 677^{+56}_{-61}\,\text{MeV}, \quad m_t(m_Z) = 181 \pm 13\,\text{GeV}, \\
m_d(m_Z) &= 4.69^{+0.60}_{-0.66}\,\text{MeV}, \quad m_s(m_Z) = 93.4^{+11.8}_{-13.0}\,\text{MeV}, \quad m_b(m_Z) = 3.00 \pm 0.11\,\text{GeV}.
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
m_u(M_X) &= 1.04^{+0.19}_{-0.20}\,\text{MeV}, \quad m_c(M_X) = 302^{+25}_{-27}\,\text{MeV}, \quad m_t(M_X) = 129^{+196}_{-40}\,\text{GeV}, \\
m_d(M_X) &= 1.33^{+0.17}_{-0.19}\,\text{MeV}, \quad m_s(M_X) = 26.5^{+3.3}_{-3.7}\,\text{MeV}, \quad m_b(M_X) = 1.00 \pm 0.04\,\text{GeV}.
\end{aligned}
\tag{3.2}
$$

The observed quark mixing angles and the CP violating phase at $\mu = m_Z$ are given by [13]

$$
\begin{aligned}
\sin\theta_{12}(m_Z) &= 0.2243 \pm 0.0016, \quad \sin\theta_{23}(m_Z) = 0.0413 \pm 0.0015, \\
\sin\theta_{13}(m_Z) &= 0.0037 \pm 0.0005, \quad \delta(m_Z) = 60° \pm 14°.
\end{aligned}
\tag{3.3}
$$

Using the values in Eq. (3.3) as inputs, we obtain the following numerical values at $\mu = M_X$:

$$\sin\theta_{12}^0 = 0.2226 - 0.2259, \quad \sin\theta_{23}^0 = 0.0295 - 0.0383,$$

$$\sin\theta_{13}^0 = 0.0024 - 0.0038, \quad \delta^0 = 46° - 74°, \tag{3.4}$$

$$|V^0| = \begin{pmatrix} 0.9741 - 0.9749 & 0.2226 - 0.2259 & 0.0024 - 0.0038 \\ 0.2225 - 0.2259 & 0.9734 - 0.9745 & 0.0295 - 0.0387 \\ 0.0048 - 0.0084 & 0.0289 - 0.0379 & 0.9993 - 0.9996 \end{pmatrix}. \tag{3.5}$$

The ratios among CKM matrix elements are

$$|V_{cd}^0/V_{cs}^0| = 0.2284 - 0.2320, \qquad |V_{us}^0/V_{cs}^0| = 0.2285 - 0.2321,$$

$$|V_{td}^0/V_{ts}^0| = 0.1699 - 0.2252, \qquad |V_{td}^0/V_{tb}^0| = 0.0050 - 0.0084,$$

$$|V_{ub}^0/V_{cb}^0| = 0.0747 - 0.1055, \qquad |V_{ub}^0/V_{tb}^0| = 0.0024 - 0.0038. \tag{3.6}$$

These values are useful to derive the allowed regions in the parameters α_u and α_d as shown in next section.

IV. CKM QUARK MIXING MATRIX

In our model, M_d and M_u have the same zero texture with same or different assignments as follows:

$$M_d = \begin{pmatrix} 0 & a_d e^{-i\phi_d} & a_d \\ a_d' e^{-i\phi_d} & b_d e^{-2i\phi_d} & (1 - \xi_d)b_d \\ a_d' & (1 - \xi_d)b_d & b_d \end{pmatrix}, \tag{4.1}$$

$$M_u = \begin{pmatrix} 0 & a_u e^{-i\phi_u} & a_u \\ a_u' e^{-i\phi_u} & b_u e^{-2i\phi_u} & (1 - \xi_u)b_u \\ a_u' & (1 - \xi_u)b_u & b_u \end{pmatrix}, \tag{4.2}$$

where ϕ_d and ϕ_u are phase parameters. We analyze the CKM quark mixing matrix of the model by taking the type A, the type B, and the type C assignments for up and down quarks. Now we present analytical results for only following case (i), case (ii), and case (iii), since the results for other cases are similarly obtained:

Case (i): Type A assignment for up quarks and type B for down quarks are taken.

Case (ii): Type B assignment for up quarks and type A for down quarks are taken.

Case (iii): Type A assignment both for up quarks and down quarks are taken.

As shown later, only case (i) provides reasonable predictions for the CKM quark mixing matrix which are consistent with Eq. (3.4). For other cases we have no consistent predictions if we use the center values of running quark masses.

In the following discussions, we denote the quark masses (m_{1f}, m_{2f}, m_{3f}) as (m_u, m_c, m_t) for $f = u$, and as (m_d, m_s, m_b) for $f = d$.

A. Type A for up × Type B for down

In case (i), the CKM quark mixing matrix V is given by

$$V = U_{Lu}^\dagger U_{Ld} = O_u^T P_u P_d^\dagger O_d' = O_u^T P O_d'$$

$$= \begin{pmatrix} c_u c_d' + \rho s_u s_d' & \sigma s_u & c_u s_d' - \rho s_u c_d' \\ s_u c_d' - \rho c_u s_d' & -\sigma c_u & s_u s_d' + \rho c_u c_d' \\ -\sigma s_d' & -\rho & \sigma c_d' \end{pmatrix} \equiv V^0, \tag{4.3}$$

where

$$s_u = \sqrt{\frac{\frac{|m_u| m_c}{\alpha_u} - m_u^2}{m_c^2 - m_u^2}}, \qquad\qquad c_u = \sqrt{\frac{m_c^2 - \frac{|m_u| m_c}{\alpha_u}}{m_c^2 - m_u^2}},$$

$$s_d' = \sqrt{\frac{\frac{|m_d| m_b}{\alpha_d} - m_d^2}{m_b^2 - m_d^2}}, \qquad\qquad c_d' = \sqrt{\frac{m_b^2 - \frac{|m_d| m_b}{\alpha_d}}{m_b^2 - m_d^2}}. \tag{4.4}$$

Here we have put

$$P \equiv P_u P_d^\dagger \equiv \mathrm{diag}(1, e^{i\delta_2}, e^{i\delta_3}), \tag{4.5}$$

and O_u and O_d' are shown in Eq. (2.30) and Eq. (2.34), respectively. The ρ and σ are defined by

$$\rho = \frac{1}{2}(e^{i\delta_3} + e^{i\delta_2}) = \cos\frac{\delta_3 - \delta_2}{2} \exp i\left(\frac{\delta_3 + \delta_2}{2}\right), \tag{4.6}$$

$$\sigma = \frac{1}{2}(e^{i\delta_3} - e^{i\delta_2}) = \sin\frac{\delta_3 - \delta_2}{2} \exp i\left(\frac{\delta_3 + \delta_2}{2} + \frac{\pi}{2}\right). \tag{4.7}$$

By using Eqs. (4.5) and (2.6), the phases δ_2 and δ_3 in our model are given by

$$\delta_2 = \varphi_d - \varphi_u - (\phi_d - \phi_u), \tag{4.8}$$

$$\delta_3 = \varphi_d - \varphi_u. \tag{4.9}$$

Note also that we obtain phase-parameter-independent relations,

$$\frac{|V_{us}^0|}{|V_{cs}^0|} = \frac{s_u}{c_u} = \sqrt{\frac{|m_u|}{m_c}}\sqrt{\frac{\frac{m_c}{\alpha_u} - |m_u|}{m_c - \frac{|m_u|}{\alpha_u}}}, \tag{4.10}$$

$$\frac{|V_{td}^0|}{|V_{tb}^0|} = \frac{s_d'}{c_d'} = \sqrt{\frac{|m_d|}{m_b}}\sqrt{\frac{\frac{m_b}{\alpha_d} - |m_d|}{m_b - \frac{|m_d|}{\alpha_d}}}. \tag{4.11}$$

Therefore, from Eqs. (4.10) and (4.11) with Eq. (3.6), we can fix the parameters α_u and α_d as

$$\alpha_u = \frac{\left|\frac{V_{us}^0}{V_{cs}^0}\right|^2 + \frac{m_u^2}{m_c^2}}{\frac{|m_u|}{m_c}\left(1 + \left|\frac{V_{us}^0}{V_{cs}^0}\right|^2\right)} = +0.0674 \sim +0.0694 \ (+0.0503 \sim +0.0901), \tag{4.12}$$

$$\alpha_d = \frac{\left|\frac{V_{td}^0}{V_{tb}^0}\right|^2 + \frac{m_d^2}{m_b^2}}{\frac{|m_d|}{m_b}\left(1 + \left|\frac{V_{td}^0}{V_{tb}^0}\right|^2\right)} = +18.3303 \sim +50.1285 \ (+15.2264 \sim +57.4358). \tag{4.13}$$

Here, the values out of the parentheses are derived by using the center values of quark masses in Eq. (3.2). On the other hand, the values in the parentheses are obtained by using quark mass values in the ranges with the estimation errors in Eq. (3.2).

The phases ϕ_u, φ_u, ϕ_d, and φ_d are given by

$$\cos\phi_u = \frac{|X_u|^2 - m_t^2}{\sqrt{(|X_u|^2 + m_t^2)^2 - 4m_t^2|X_u|^2\cos^2\eta_u}}, \tag{4.14}$$

$$\cos\phi_d = \frac{|X_d|^2 - m_s^2}{\sqrt{(|X_d|^2 + m_s^2)^2 - 4m_s^2|X_d|^2\cos^2\eta_d}}, \tag{4.15}$$

$$\cos\varphi_u = \frac{|X_u| - m_t\cos\eta_u}{\sqrt{|X_u|^2 + m_t^2 - 2m_t|X_u|\cos\eta_u}}, \tag{4.16}$$

$$\cos\varphi_d = \frac{|X_d| - m_s\cos\eta_d}{\sqrt{|X_d|^2 + m_s^2 - 2m_s|X_d|\cos\eta_d}}, \tag{4.17}$$

where $|X_u|^2$ and $|X_d|^2$ are given by

$$|X_u|^2 = m_u^2 + m_c^2 - |m_u|m_c\left(\frac{1 + \alpha_u^2}{\alpha_u}\right), \tag{4.18}$$

$$|X_d|^2 = m_d^2 + m_b^2 - |m_d|m_b\left(\frac{1 + \alpha_d^2}{\alpha_d}\right). \tag{4.19}$$

Therefore, we obtain

$$\phi_u = +3.1370 \sim +3.1462 \ (+3.1344 \sim +3.1488), \tag{4.20}$$

$$\phi_d = -0.0548 \sim +0.0548 \ (-0.0650 \sim +0.0650), \tag{4.21}$$

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from Eqs. (4.14) – (4.15) with use of the restrictions in Eqs. (4.12)–(4.13) and free η_u and η_d (i.e. $0 \leq \eta_u \leq 2\pi$ and $0 \leq \eta_d \leq 2\pi$). It should be noted that all components of V are dependent on δ_3, δ_2, α_u, and α_d. The phase parameters δ_3 and δ_2 are dependent on α_u, α_d, η_u, and η_d. Therefore, the present model has four adjustable parameters, α_u, α_d, η_u, and η_d. Then, the explicit magnitudes of the components of V are obtained as

$$\left|V_{cs}^0\right| = |\sigma c_u| = \left|c_u \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.9737 - 0.9749 \, (0.9539 - 0.9852), \quad (4.22)$$

$$\left|V_{us}^0\right| = |\sigma s_u| = \left|s_u \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.2226 - 0.2260 \, (0.1687 - 0.2981), \quad (4.23)$$

$$\left|V_{td}^0\right| = |\sigma s_d'| = \left|s_d' \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0050 - 0.0084 \, (0.0042 - 0.0100), \quad (4.24)$$

$$\left|V_{ts}^0\right| = |\rho| = \left|\cos\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0297 \, (0.0000 - 0.0361), \quad (4.25)$$

$$\left|V_{tb}^0\right| = |\sigma c_d'| = \left|c_d' \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.9995 - 1.0000 \, (0.9993 - 1.0000). \quad (4.26)$$

By using these values out of the parentheses which are obtained by using the center values of quark masses in Eq. (3.2), the quark mixing angles are predicted as follows:

$$\sin\theta_{12}^0 = 0.2226 - 0.2260, \quad \sin\theta_{23}^0 = 0.0000 - 0.0303, \quad \sin\theta_{13}^0 = 0.0000 - 0.0137. \quad (4.27)$$

These predicted values are consistent with Eq. (3.4).

By using the rephasing of the up and down quarks, Eq. (4.3) is changed to the standard representation of the CKM quark mixing matrix,

$$
\begin{aligned}
V_{\text{std}} &= \text{diag}(e^{i\zeta_1^u}, e^{i\zeta_2^u}, e^{i\zeta_2^u}) \, V \, \text{diag}(e^{i\zeta_1^d}, e^{i\zeta_2^d}, e^{i\zeta_2^d}) \\
&= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} .
\end{aligned}
\quad (4.28)
$$

Here ζ_i^q comes from the rephasing in the quark fields to make the choice of phase convention. The CP violating phase δ^0 in Eq. (4.28) is predicted with the expression of V^0 in Eq. (4.3) as

$$\delta^0 = \arg\left[\left(\frac{V_{us}^0 V_{cs}^{0*}}{V_{ub}^0 V_{cb}^{0*}}\right) + \frac{|V_{us}^0|^2}{1 - |V_{ub}^0|^2}\right] = 0 - 2\pi . \quad (4.29)$$

So far we have predicted magnitudes for the components of $|V_{ij}^0|$ and δ^0 by using Eqs. (4.12)–(4.13) and free parameters η_u and η_d (i.e. $0 \leq \eta_u \leq 2\pi$ and $0 \leq \eta_d \leq 2\pi$).

Conversely, let us derive the allowed regions of the parameters, α_u, α_d, η_u, and η_d from the experimental restriction of Eqs. (3.4) - (3.6). In Fig. 2, we present the allowed regions of the parameters, α_u, α_d, η_u, and η_d in which the experimental restrictions of Eqs. (3.4) - (3.6) are satisfied simultaneously. In Fig. 3, with use of free η_u and η_d, we also obtain the allowed regions in the $\alpha_u - \alpha_d$ parameter space which come from the restrictions of $|V_{us}^0|$, $|V_{cb}^0|$, $|V_{ub}^0|$, and the CP violating phase δ^0 in Eqs. (3.4) - (3.5), respectively. As seen from Fig. 2 and Fig. 3, the case (i) is well consistent with the observed experimental data.

B.   Type B for up $\times$ Type A for down

In case (ii), the CKM quark mixing matrix V is given by

$$V = U_{Lu}^\dagger U_{Ld} = O_u'^T P_u P_d^\dagger O_d = O_u'^T P O_d$$

$$= \begin{pmatrix} c_u' c_d + \rho s_u' s_d & c_u' s_d - \rho s_u' c_d & -\sigma s_u' \\ \sigma s_d & -\sigma c_d & -\rho \\ s_u' c_d - \rho c_u' s_d & s_u' s_d + \rho c_u' c_d & \sigma c_u' \end{pmatrix} \equiv V^0, \tag{4.30}$$

where

$$s_u' = \sqrt{\frac{\frac{|m_u| m_t}{\alpha_u} - m_u^2}{m_t^2 - m_u^2}}, \qquad c_u' = \sqrt{\frac{m_t^2 - \frac{|m_u| m_t}{\alpha_u}}{m_t^2 - m_u^2}},$$

$$s_d = \sqrt{\frac{\frac{|m_d| m_s}{\alpha_d} - m_d^2}{m_s^2 - m_d^2}}, \qquad c_d = \sqrt{\frac{m_s^2 - \frac{|m_d| m_s}{\alpha_d}}{m_s^2 - m_d^2}}, \tag{4.31}$$

and O_d and O_u' are shown in Eq. (2.30) and Eq. (2.34), respectively. Using the phase-parameter-independent relations,

$$\frac{|V_{ub}^0|}{|V_{tb}^0|} = \frac{s_u'}{c_u'} = \sqrt{\frac{|m_u|}{m_t}} \sqrt{\frac{\frac{m_t}{\alpha_u} - |m_u|}{m_t - \frac{|m_u|}{\alpha_u}}}, \tag{4.32}$$

$$\frac{|V_{cd}^0|}{|V_{cs}^0|} = \frac{s_d}{c_d} = \sqrt{\frac{|m_d|}{m_s}} \sqrt{\frac{\frac{m_s}{\alpha_d} - |m_d|}{m_s - \frac{|m_d|}{\alpha_d}}}, \tag{4.33}$$

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with Eq. (3.6), we obtain

$$\alpha_u = \frac{\left|\frac{V_{ub}^0}{V_{tb}^0}\right|^2 + \frac{m_u^2}{m_t^2}}{\frac{|m_u|}{m_t}\left(1 + \left|\frac{V_{ub}^0}{V_{tb}^0}\right|^2\right)} = +0.5706 \sim +1.4294 \ (+0.1829 \sim +2.4503), \qquad (4.34)$$

$$\alpha_d = \frac{\left|\frac{V_{cd}^0}{V_{cs}^0}\right|^2 + \frac{m_d^2}{m_s^2}}{\frac{|m_d|}{m_s}\left(1 + \left|\frac{V_{cd}^0}{V_{cs}^0}\right|^2\right)} = +0.9388 \sim +0.9658 \ (+0.7293 \sim +1.2255). \qquad (4.35)$$

The phases ϕ_u, φ_u, ϕ_d, and φ_d are given by

$$\cos\phi_u = \frac{|X_u|^2 - m_c^2}{\sqrt{(|X_u|^2 + m_c^2)^2 - 4m_c^2|X_u|^2\cos^2\eta_u}}, \qquad (4.36)$$

$$\cos\phi_d = \frac{|X_d|^2 - m_b^2}{\sqrt{(|X_d|^2 + m_b^2)^2 - 4m_b^2|X_d|^2\cos^2\eta_d}}, \qquad (4.37)$$

$$\cos\varphi_u = \frac{|X_u| - m_c\cos\eta_u}{\sqrt{|X_u|^2 + m_c^2 - 2m_c|X_u|\cos\eta_u}}, \qquad (4.38)$$

$$\cos\varphi_d = \frac{|X_d| - m_b\cos\eta_d}{\sqrt{|X_d|^2 + m_b^2 - 2m_b|X_d|\cos\eta_d}}, \qquad (4.39)$$

where

$$|X_u|^2 = m_u^2 + m_t^2 - |m_u|m_t\left(\frac{1+\alpha_u^2}{\alpha_u}\right), \qquad (4.40)$$

$$|X_d|^2 = m_d^2 + m_s^2 - |m_d|m_s\left(\frac{1+\alpha_d^2}{\alpha_d}\right). \qquad (4.41)$$

Therefore, we obtain

$$\phi_u = -0.0047 \sim +0.0047 \ (-0.0073 \sim +0.0073), \qquad (4.42)$$

$$\phi_d = +3.0913 \sim +3.1919 \ (+3.0819 \sim +3.2013), \qquad (4.43)$$

from Eqs. (4.36)–(4.37) with use of the restrictions of α_u and α_d in Eqs. (4.34)–(4.35) and free η_u and η_d (i.e. $0 \le \eta_u \le 2\pi$ and $0 \le \eta_d \le 2\pi$). Hence, the explicit magnitudes of the

components of V are obtained as

$$\left|V_{cb}^0\right| = |\rho| = \left|\cos\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0275 \,(0.0000 - 0.0335), \qquad (4.44)$$

$$\left|V_{ub}^0\right| = |\sigma s_u'| = \left|\sin\left(\frac{\delta_3 - \delta_2}{2}\right)s_u'\right| = 0.0024 - 0.0038 \,(0.0010 - 0.0087), \qquad (4.45)$$

$$\left|V_{cd}^0\right| = |\sigma s_d| = \left|\sin\left(\frac{\delta_3 - \delta_2}{2}\right)s_d\right| = 0.2226 - 0.2260 \,(0.1725 - 0.2937), \qquad (4.46)$$

$$\left|V_{cs}^0\right| = |\sigma c_d| = \left|\sin\left(\frac{\delta_3 - \delta_2}{2}\right)c_d\right| = 0.9738 - 0.9749 \,(0.9556 - 0.9850), \qquad (4.47)$$

$$\left|V_{tb}^0\right| = |\sigma c_u'| = \left|\sin\left(\frac{\delta_3 - \delta_2}{2}\right)c_u'\right| = 0.9996 - 0.9997 \,(0.9994 - 0.9997). \qquad (4.48)$$

Here the values in parentheses, which are obtained by using the quark masses in the ranges with the estimation errors in Eq. (3.2), are consistent with Eq. (3.5). However, the values out of parentheses are not. Namely, when we use the center values of quark masses in Eq. (3.2), the quark mixing angles are predicted as

$$\sin\theta_{12}^0 = 0.2225 - 0.2260, \qquad\qquad \sin\theta_{23}^0 = 0.0000 - 0.0275,$$

$$\sin\theta_{13}^0 = 0.0024 - 0.0038, \qquad\qquad \delta^0 = -3.0896 - +3.0841, \qquad (4.49)$$

which are inconsistent with Eq. (3.4). In Fig. 4, allowed regions in the $\alpha_u - \alpha_d$ parameter space are derived, which are obtained from the restrictions of $|V_{us}^0|$, $|V_{cb}^0|$, $|V_{ub}^0|$, and the CP violating phase δ^0 in Eqs. (3.4) - (3.5), respectively with use of free η_u and η_d (i.e. $0 \le \eta_u \le 2\pi$ and $0 \le \eta_d \le 2\pi$). As seen from Fig. 4, the case (ii) is inconsistent with Eq. (3.5) if we use the center values of the quark masses.

C. Type A for up $\times$ Type A for down

In case (iii), the CKM quark mixing matrix V is given by

$$V = U_{Lu}^\dagger U_{Ld} = O_u^T P_u P_d^\dagger O_d$$

$$= \begin{pmatrix} c_u c_d + \rho s_u s_d & c_u s_d - \rho s_u c_d & -\sigma s_u \\ s_u c_d - \rho c_u s_d & s_u s_d + \rho c_u c_d & \sigma c_u \\ -\sigma s_d & \sigma c_d & \rho \end{pmatrix} \equiv V^0, \qquad (4.50)$$

where

$$s_u = \sqrt{\frac{\frac{|m_u|m_c}{\alpha_u} - m_u^2}{m_c^2 - m_u^2}}, \qquad\qquad c_u = \sqrt{\frac{m_c^2 - \frac{|m_u|m_c}{\alpha_u}}{m_c^2 - m_u^2}},$$

$$s_d = \sqrt{\frac{\frac{|m_d|m_s}{\alpha_d} - m_d^2}{m_s^2 - m_d^2}}, \qquad\qquad c_d = \sqrt{\frac{m_s^2 - \frac{|m_d|m_s}{\alpha_d}}{m_s^2 - m_d^2}}, \qquad (4.51)$$

and O_d (O_u) is shown in Eq. (2.30). Here we have put $P \equiv P_u P_d^\dagger \equiv \mathrm{diag}(1, e^{i\delta_2}, e^{i\delta_3})$. The ρ and σ are defined by Eqs. (4.6) and (4.7), respectively. Using the phase-parameter-independent relations,

$$\frac{|V_{ub}^0|}{|V_{cb}^0|} = \frac{s_u}{c_u} = \sqrt{\frac{|m_u|}{m_c}} \sqrt{\frac{\frac{m_c}{\alpha_u} - |m_u|}{m_c - \frac{|m_u|}{\alpha_u}}}, \qquad (4.52)$$

$$\frac{|V_{td}^0|}{|V_{ts}^0|} = \frac{s_d}{c_d} = \sqrt{\frac{|m_d|}{m_s}} \sqrt{\frac{\frac{m_s}{\alpha_d} - |m_d|}{m_s - \frac{|m_d|}{\alpha_d}}}, \qquad (4.53)$$

with Eq. (3.6), we obtain

$$\alpha_u = \frac{\left|\frac{V_{ub}^0}{V_{cb}^0}\right|^2 + \frac{m_u^2}{m_c^2}}{\frac{|m_u|}{m_c}\left(1 + \left|\frac{V_{ub}^0}{V_{cb}^0}\right|^2\right)} = +0.3123 \sim +0.6188 \ (+0.2330 \sim +0.8026), \qquad (4.54)$$

$$\alpha_d = \frac{\left|\frac{V_{td}^0}{V_{ts}^0}\right|^2 + \frac{m_d^2}{m_s^2}}{\frac{|m_d|}{m_s}\left(1 + \left|\frac{V_{td}^0}{V_{ts}^0}\right|^2\right)} = +0.9907 \sim +1.6458 \ (+0.7704 \sim +2.0398). \qquad (4.55)$$

The phases ϕ_u, φ_u, ϕ_d, and φ_d are given by

$$\cos\phi_u = \frac{|X_u|^2 - m_t^2}{\sqrt{(|X_u|^2 + m_t^2)^2 - 4m_t^2|X_u|^2\cos^2\eta_u}}, \qquad (4.56)$$

$$\cos\phi_d = \frac{|X_d|^2 - m_b^2}{\sqrt{(|X_d|^2 + m_b^2)^2 - 4m_b^2|X_d|^2\cos^2\eta_d}}, \qquad (4.57)$$

$$\cos\varphi_u = \frac{|X_u| - m_t\cos\eta_u}{\sqrt{|X_u|^2 + m_t^2 - 2m_t|X_u|\cos\eta_u}}, \qquad (4.58)$$

$$\cos\varphi_d = \frac{|X_d| - m_b\cos\eta_d}{\sqrt{|X_d|^2 + m_b^2 - 2m_b|X_d|\cos\eta_d}}, \qquad (4.59)$$

where

$$|X_u|^2 = m_u^2 + m_c^2 - |m_u|m_c\left(\frac{1 + \alpha_u^2}{\alpha_u}\right), \qquad (4.60)$$

$$|X_d|^2 = m_d^2 + m_s^2 - |m_d|m_s\left(\frac{1 + \alpha_d^2}{\alpha_d}\right). \qquad (4.61)$$

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Therefore, we obtain

$$\phi_u = +3.1369 \sim +3.1463 \ (+3.1343 \sim +3.1489), \tag{4.62}$$

$$\phi_d = +3.0913 \sim +3.1919 \ (+3.0819 \sim +3.2013), \tag{4.63}$$

from Eqs. (4.56)–(4.57) with use of the restrictions of α_u and α_d in Eqs. (4.54)–(4.55) and free η_u and η_d (i.e. $0 \leq \eta_u \leq 2\pi$ and $0 \leq \eta_d \leq 2\pi$). Hence, the explicit magnitudes of the components of V are obtained as

$$\left|V_{cb}^0\right| = |\sigma c_u| = \left|c_u \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0274 \ (0.0000 - 0.0334), \tag{4.64}$$

$$\left|V_{ub}^0\right| = |\sigma s_u| = \left|s_u \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0029 \ (0.0000 - 0.0046), \tag{4.65}$$

$$\left|V_{td}^0\right| = |\sigma s_d| = \left|s_d \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0060 \ (0.0000 - 0.0083), \tag{4.66}$$

$$\left|V_{ts}^0\right| = |\sigma c_d| = \left|c_d \sin\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.0000 - 0.0269 \ (0.0000 - 0.0329), \tag{4.67}$$

$$\left|V_{tb}^0\right| = |\rho| = \left|\cos\left(\frac{\delta_3 - \delta_2}{2}\right)\right| = 0.9996 - 1.0000 \ (0.9994 - 1.0000). \tag{4.68}$$

Here the values in parentheses, which are obtained by using the quark mass values in the ranges with the estimation errors in Eq. (3.2), are consistent with Eq. (3.5). However, the values out of parentheses are not. Namely, when we use the center values of quark masses in Eq. (3.2), the quark mixing angles are predicted as

$$\sin\theta_{12}^0 = 0.0631 - 0.3208, \qquad\qquad \sin\theta_{23}^0 = 0.0000 - 0.0274,$$

$$\sin\theta_{13}^0 = 0.0000 - 0.0029, \qquad\qquad\quad \delta^0 = 0 - 2\pi, \tag{4.69}$$

which are inconsistent with Eq. (3.4). In Fig. 5, allowed regions in the $\alpha_u - \alpha_d$ parameter space are derived, which are obtained from the restrictions of $|V_{us}^0|$, $|V_{cb}^0|$, $|V_{ub}^0|$, and the CP violating phase δ^0 in Eqs. (3.4) - (3.5), respectively with use of free η_u and η_d (i.e. $0 \leq \eta_u \leq 2\pi$ and $0 \leq \eta_d \leq 2\pi$). As seen from Fig. 5, the case (iii) is inconsistent with Eq. (3.5) if we use the center values of the quark masses.

We have presented the results for only the case (i), case (ii), and case (iii). For other possible cases we have no consistent allowed regions of the parameters.

V. CONCLUSION

We have proposed a mass matrix model that gives a universal description of quark and lepton with the same texture form (2.1), based on a flavor $2 \leftrightarrow 3$ symmetry including a phase ϕ. Against the conventional picture that the mass matrix forms in the quark sector will take somewhat different structures from those in the lepton sector, we have considered a scenario in which the flavor $2 \leftrightarrow 3$ symmetry play a role in the quark mass matrices as well as in the lepton mass matrices as a fundamental common symmetry in the Yukawa couplings. By using the quark masses as inputs, the present model has four adjustable parameters, α_u, α_d, η_u, and η_d to reproduce the observed CKM quark mixing matrix parameters at the electroweak scale. We have shown that the observed small CKM quark mixings are realized by a fine tuning of the parameters in the case (i) where type A assignment for up quarks and type B for down quarks are taken. With this consistent parameters, our result suggests an interesting mass matrix structure for up and down quarks, which will give a clue to a possible form of mass matrix. Since the components of each mass matrix M_f takes different values, the present model cannot be embedded into a GUT scenario. However, it is worthwhile noting that it can give a universal description of quark and lepton mass matrices with the same texture form based on a flavor $2 \leftrightarrow 3$ symmetry.

APPENDIX

Following the discussions in Ref. [14], let us give a brief review of the diagonalization of mass matrix $\widehat{M}$ given by

$$\widehat{M} = \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix}. \tag{A.1}$$

The eigenvalues of $\widehat{M}$ are given by

$$- \lambda_{1f} = \frac{1}{2} \left[B_f + C_f - \sqrt{8A_f^2 + (B_f + C_f)^2} \right], \tag{A.2}$$

$$\lambda_{2f} = \frac{1}{2} \left[B_f + C_f + \sqrt{8A_f^2 + (B_f + C_f)^2} \right], \tag{A.3}$$

$$\lambda_{3f} = B_f - C_f. \tag{A.4}$$

To the contrary, the texture's components A_f, B_f, and C_f are expressed in terms of λ_{if} as

$$\begin{aligned}
A_f &= \sqrt{\frac{\lambda_{2f}\lambda_{1f}}{2}}, \\
B_f &= \frac{1}{2} \left(\lambda_{3f} + \lambda_{2f} - \lambda_{1f} \right), \\
C_f &= -\frac{1}{2} \left(\lambda_{3f} - \lambda_{2f} + \lambda_{1f} \right).
\end{aligned} \tag{A.5}$$

Here, there are following three types of assignments for the eigenvalues λ_{if}:

(i) Type A:

In this type, the mass eigenvalues λ_{1f}, λ_{2f}, and λ_{3f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $\lambda_{1f} < \lambda_{2f} < \lambda_{3f}$.) Therefore, $\widehat{M}$ is diagonalized by an orthogonal matrix O_f as

$$O_f^T \widehat{M} O_f = \mathrm{diag}(-\lambda_{1f}, \lambda_{2f}, \lambda_{3f}), \tag{A.6}$$

with

$$O_f \equiv \begin{pmatrix} c_f & s_f & 0 \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{A.7}$$

where

$$c_f = \sqrt{\frac{\lambda_{2f}}{\lambda_{2f} + \lambda_{1f}}}, \quad s_f = \sqrt{\frac{\lambda_{1f}}{\lambda_{2f} + \lambda_{1f}}}. \tag{A.8}$$

(ii) Type B:

In this type, the mass eigenvalues λ_{1f}, λ_{3f}, and λ_{2f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $\lambda_{1f} < \lambda_{3f} < \lambda_{2f}$.) Therefore, $\widehat{M}$ is diagonalized by an orthogonal matrix O_f' as

$$O_f'^T \widehat{M} O_f' = \mathrm{diag}(-\lambda_{1f}, \lambda_{3f}, \lambda_{2f}), \tag{A.9}$$

with

$$O'_f \equiv \begin{pmatrix} c_f & 0 & s_f \\ -\frac{s_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}. \tag{A.10}$$

Here O'_f is obtained from O_f by exchanging the second row for the third one.

(iii) Type C:

In this type, the mass eigenvalues λ_{3f}, λ_{1f}, and λ_{2f} are allocated to the masses of the first, second, and third generations, respectively. (i.e. $\lambda_{3f} < \lambda_{1f} < \lambda_{2f}$.) Therefore, $\widehat{M}$ is diagonalized by an orthogonal matrix O''_f as

$$O''^T_f \widehat{M} O''_f = \mathrm{diag}(\lambda_{3f}, -\lambda_{1f}, \lambda_{2f}), \tag{A.11}$$

with

$$O''_f \equiv \begin{pmatrix} 0 & c_f & s_f \\ \frac{1}{\sqrt{2}} & -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}. \tag{A.12}$$

Here O''_f is obtained from O'_f by exchanging the first row for the second one.

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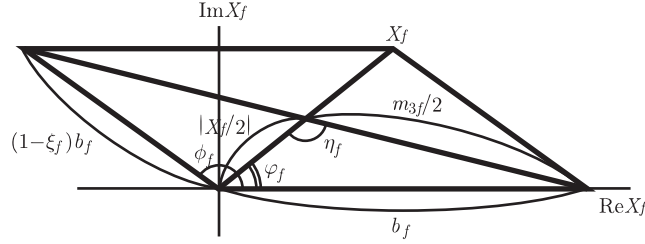[18] H. Fusaoka and Y. Koide, Phys. Rev. **D 57**, 3986 (1998).

FIG. 1: The relations among the components in the mass matrix, and the definition of a free CP violating phase parameter η_f which is independent of the mass eigenvalues.
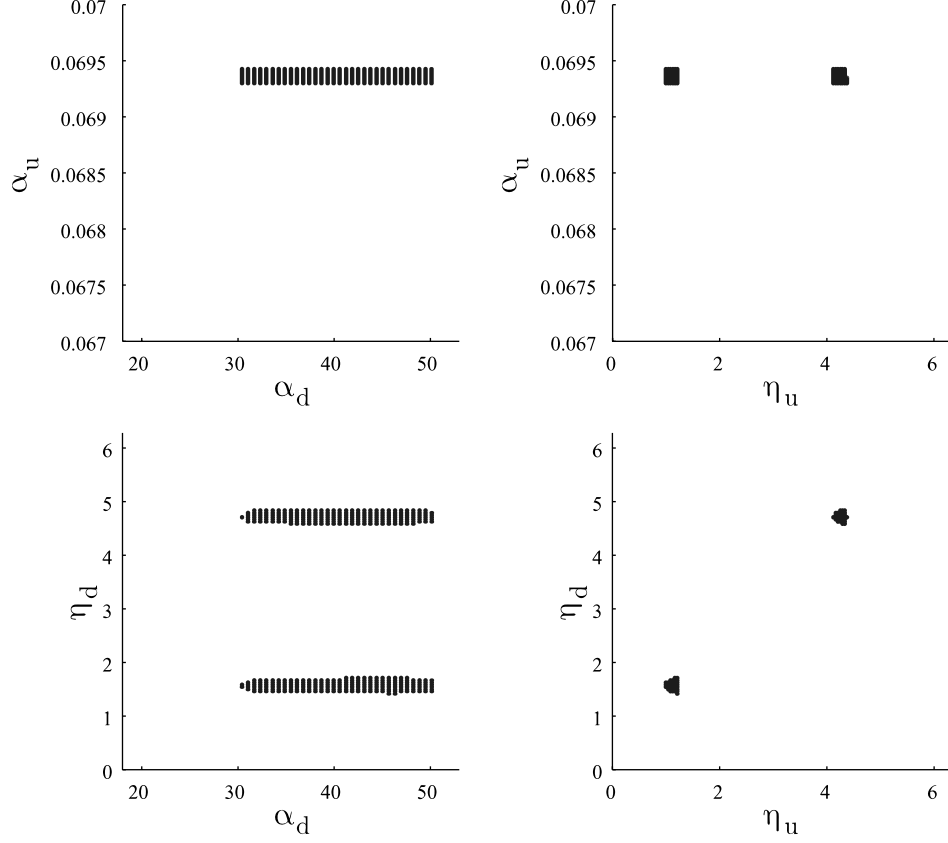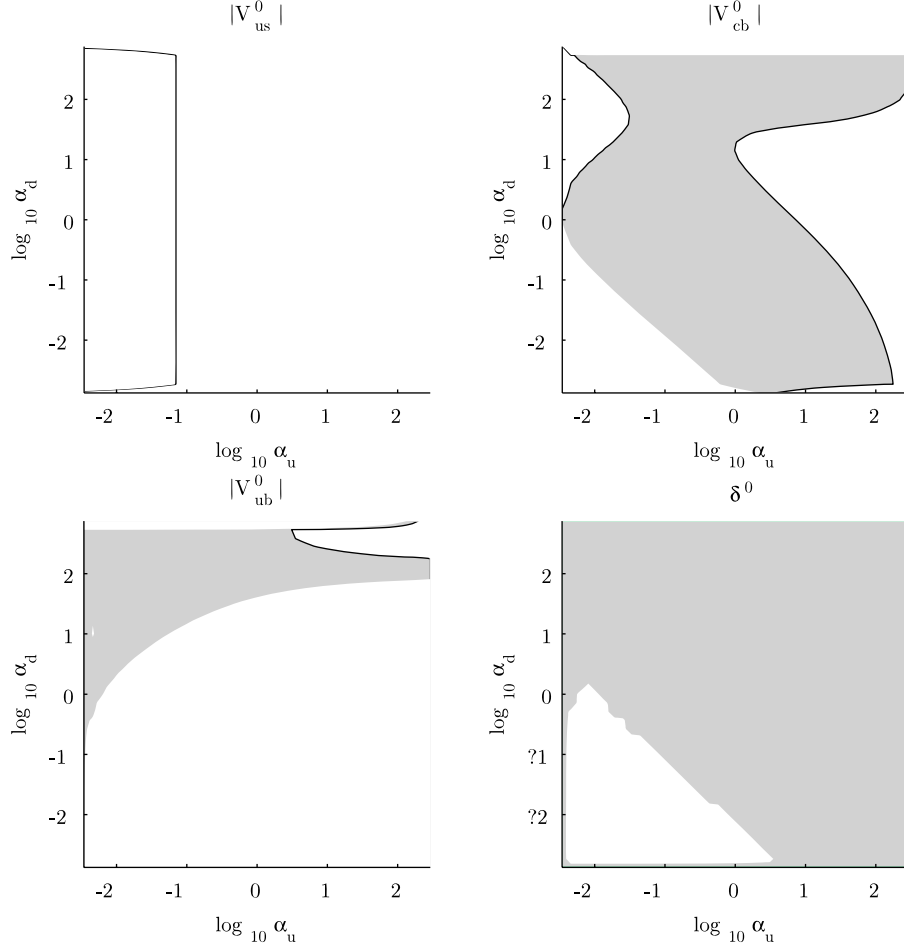
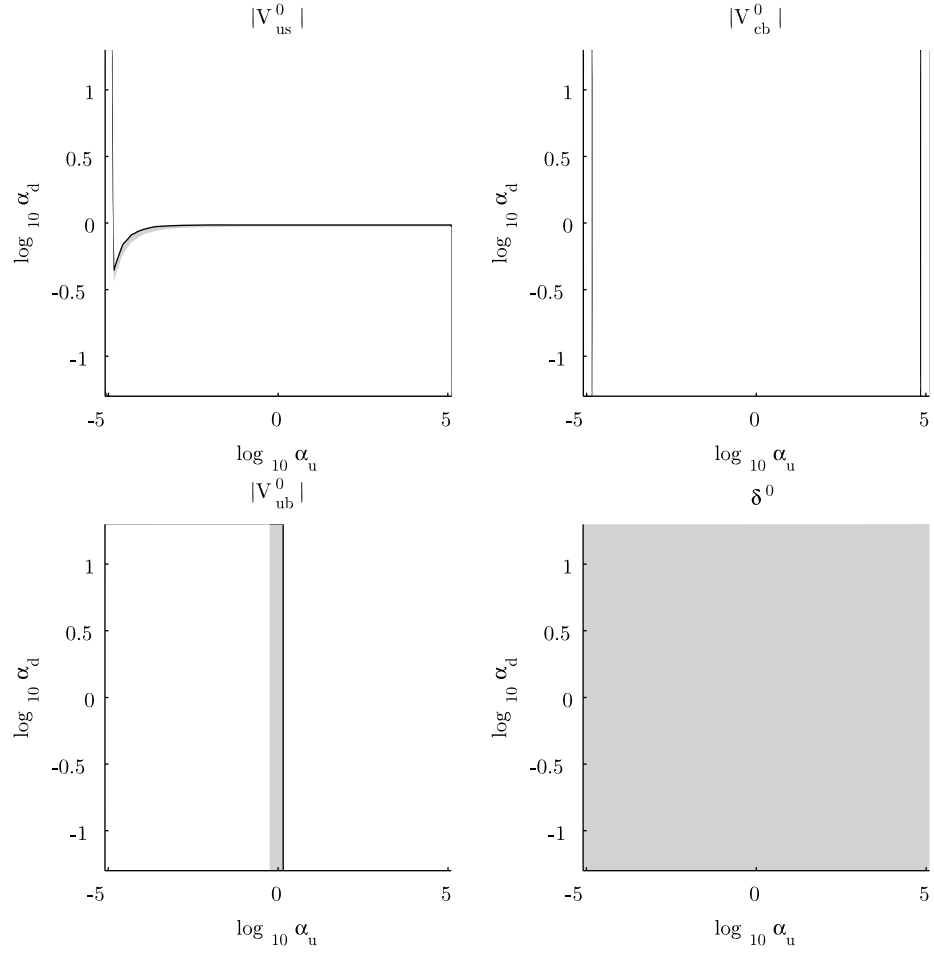FIG. 2: The allowed region in the parameter planes for the case (i) in which Type A assignment for up quarks and type B for down quarks are taken. The dots indicate the allowed regions, in which the CKM quark mixing angles $\sin\theta_{12}^0$, $\sin\theta_{23}^0$, $\sin\theta_{13}^0$, and the CP violating phase δ^0 at the GUT scale satisfy the restrictions of Eq. (3.4), simultaneously.

FIG. 3: The allowed region in the α_u - α_d parameter plane for the case (i) in which Type A assignment for up quarks and type B for down quarks are taken. The allowed regions are indicated by the dark shaded regions which are obtained from the values of the CKM quark mixing matrix elements $|V_{us}^0|$, $|V_{cb}^0|$, $|V_{ub}^0|$, and the CP violating phase δ^0 in Eqs. (3.4) - (3.5), respectively. The boundary with (without) black lines shows the upper (lower) limit. Here we take η_u and η_d as free parameters (i.e. $0 \leq \eta_u \leq 2\pi$ and $0 \leq \eta_d \leq 2\pi$).

FIG. 4: The allowed region in the α_u - α_d parameter plane for the case (ii) in which Type B assignment for up quarks and type A for down quarks are taken.
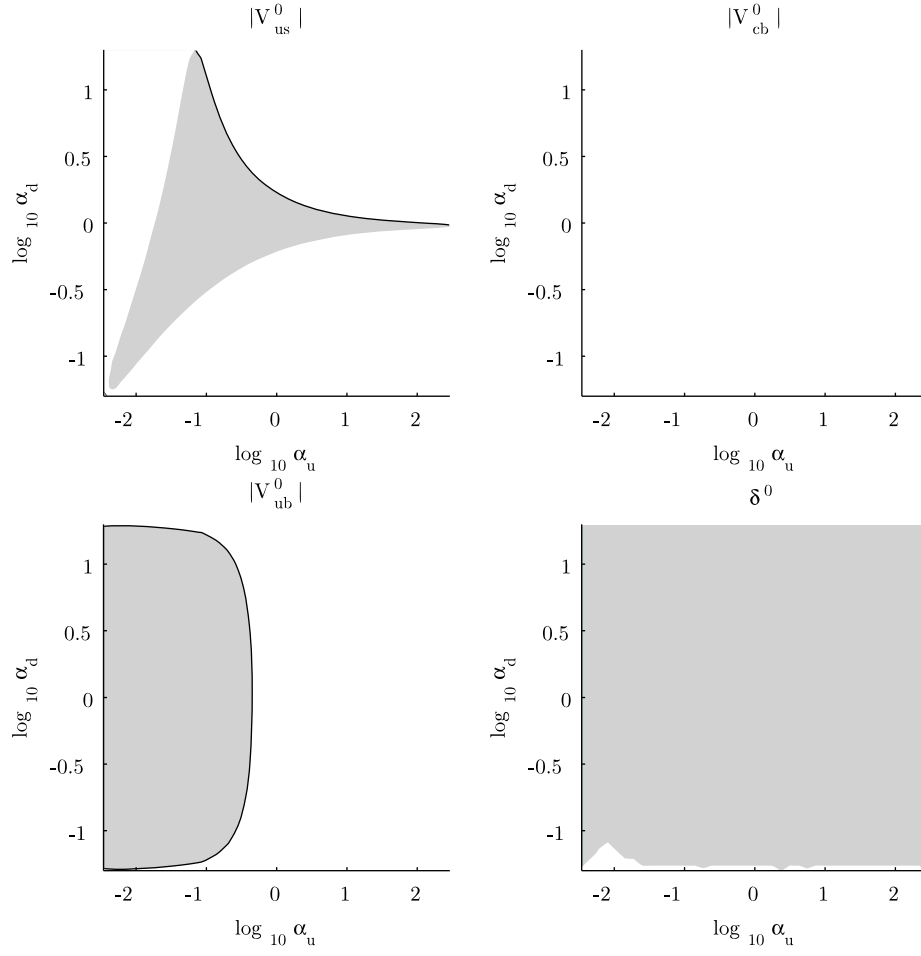
FIG. 5: The allowed region in the α_u - α_d parameter plane for the case (iii) in which Type A assignment both for up quarks and down quarks are taken.